

Membranes

Problem 1

An air-supported spherical membrane (radius = 100 ft, $\phi=90^\circ$) is subject to an internal pressure of 10 lb/ft². What is the stress in the membrane?

Uplift force: pressure x projected area = 10 x 3.14 x 100² = 314,000 lb or 314 kips

Distributed force along the length of the supporting circle:

314/(2x3.14x100) = 500 lb/ft

if thickness is t , then stress will be $500/12/t = 41.67/t$ psi

for $t=1/8$ " the stress will be: 333 psi

Problem 2

An air-inflated cylinder has a radius of 6 inches, a length of 10 ft and is subject to an internal pressure of 60 psi. What is the stress in the membrane in the radial direction? What is the stress in the membrane in the longitudinal direction? Make the necessary assumptions.

Stress in radial direction (see slide): pressure x radius = 60 x 6 = 360 lb/in

For a thickness of 1/16 inch, the stress will be: 5.760 psi

Stress in axial direction: the force at the end is pressure x area = 60 x 3.14 x 6² = 6,782 lb

The area of the cross-section is: 2 x 3.14 x 6 x 1/16 = 2.355 in²

The axial stress is force/area = 6,782/2.355 = 2,880 psi

What would be the maximum uniform load that the cylinder could support, assuming that the thickness of the membrane is 1/16 of an inch and the maximum allowable stress is 2 ksi.

Maximum moment is $wL^2/8 = 120^2w/8 = 1800w$

Moment of inertia of the cross-section is $(3.14 x 6^4)/4 - (3.14 x (6-1/16)^4)/4 = 41.73$ in⁴

$c = R = 6$ "

Bending stress: $f=Mc/I = 1800w * 6/41.73 = 258.8w$ psi

Option I: ignore the allowable stress of the membrane: bending stress should be less or equal to the axial stress due to air pressure:

258.8w = 2,880 $w = 11.13$ lb/in, or 133.5 lb/ft

Option II: since the maximum stress will be twice the axial load, i.e., $2x 2,880 = 5.760$ psi which is not allowable, it means that the air pressure and the maximum allowable load should be reduced. The maximum allowable stress is 2,000 psi, or 34.7% of the value calculated above. So both the air pressure and the uniform load must be reduced to 34.7%:

Air-pressure: 34.7% x 60 = 20.8 psi

Uniform load: 46.34 lb/ft