Appendix to Chapter 5

NONLINEAR STATIC PROCEDURE

PREFACE: This appendix addresses nonlinear static analysis, a seismic analysis procedure also sometimes known as pushover analysis, for review and comment and for adoption into a subsequent edition of the *Provisions*.

Although nonlinear static analysis has only recently been included in design provisions for new building construction, the procedure itself is not new and has been used for many years in both research and design applications. For example, nonlinear static analysis has been used for many years as a standard methodology in the design of the offshore platform structures for hydrodynamic effects and has been adopted recently in several standard methodologies for the seismic evaluation and rehabilitation of building structures, including the *Recommended Seismic Design Criteria for New Steel Moment-Frame Buildings* (FEMA-350, 2000a), *Prestandard and Commentary for the Seismic Rehabilitation of Buildings* (FEMA 356, 2000b) and *Seismic Evaluation and Retrofit of Concrete Buildings* (ATC 40, 1996). Nonlinear static analysis forms the basis for earthquake loss estimation procedures contained in *HAZUS* (NIBS, 1999), FEMA's nationally applicable earthquake loss estimation model. Although it does not explicitly appear in the *Provisions*, the nonlinear static analysis methodology also forms the basis for the equivalent lateral force procedures contained in the provisions for base-isolated structures and structures with dampers.

One of the controversies surrounding the introduction of this methodology into the *Provisions* relates to the determination of the limit deformation (sometimes called a target displacement). Several methodologies for estimating the amount of deformation induced in a structure by earthquake-induced ground shaking have been proposed and are included in various adoptions of the procedure. The approach presented in this appendix is based on statistical correlations of the displacements predicted by linear and nonlinear dynamic analyses of structures, which is similar to that contained in FEMA 356.

A second controversy relates to the limited availability of consensus-based acceptance criteria to be used to determine the adequacy of a design once the forces and deformations produced by design earthquake ground shaking are estimated. It should be noted that this limitation applies equally to the nonlinear response history approach, which already has been adopted into building codes.

Nonlinear static analysis provides a simplified method of directly evaluating nonlinear response of structures to strong earthquake ground shaking that can be an attractive alternative to the more complex procedures of nonlinear response history analysis. It is hoped that exposure of this approach

through inclusion in this appendix will allow the necessary consensus to be developed to permit later integration into the *Provisions* as such.

Users of this appendix also should consult the *Commentary* for guidance. Please direct all feedback on this appendix and its commentary to the BSSC.

A5.1 GENERAL

A5.1.1 Scope. This appendix provides guidelines for the use of the nonlinear static procedure for the analysis and design of structures.

A5.1.2 Definitions

Base: See Sec. 4.1.3.

Base shear: See Sec. 4.1.3.

Building: See Sec. 4.1.3.

Capacity curve: A plot of the total applied lateral force, V_j, versus the lateral displacement of the control point, δ_j, as determined in a nonlinear static analysis.

Component: See Sec. 1.1.4.

Control point: A point used to index the lateral displacement of the structure in a nonlinear static analysis, determined according to Sec. 5.2.1.

Dead load: See Sec. 4.1.3.

Design earthquake ground motion: See Sec. 1.1.4.

Diaphragm: See Sec. 4.1.3.

Effective Yield Displacement: The displacement of the control point at the intersection of the first and second branches of a bilinear curve that is fitted to the capacity curve according to Sec. A5.2.3.

Effective Yield Strength: The total applied lateral force at the intersection of the first and second branches of a bilinear curve that is fitted to the capacity curve according to Sec. A5.2.3.

Live load: See Sec. 4.1.3.

Registered design professional: See Sec. 2.1.3.

Seismic-force-resisting system: See Sec. 1.1.4.

Story: See Sec. 4.1.3.

Structure: See Sec. 1.1.4.

Target displacement: An estimate of the maximum expected displacement of the control point calculated for the design earthquake ground motion.

A5.1.3 Notation

C_d	See Sec. 4.1.4.
C_s	See Sec. 5.1.3
C_0	A modification factor to relate the displacement of the control point to the displacement of a representative single-degree-of-freedom system, as determined by Eq. A5.2-3.
C_1	A modification factor to account for the influence of inelastic behavior on the response of the system, as determined by Eq. A5.2-4.
g	acceleration of gravity.
j	The increment of lateral loading.
Q_E	See Sec. 4.1.4.
Q_{Ei}	individual member forces, determined according to Sec. A5.2.9.1
R	See Sec. 4.1.4.
R_d	The system ductility factor as determined by Eq. A5.2.-5.
S_a	See Sec. 3.1.4.
T_1	The fundamental period of the structure in the direction under consideration.
T_e	The effective fundamental period of the structure in the direction under consideration, as determined according to Sec. A5.2.3.
T_S	See Sec. 3.1.4.

V_j	The total applied lateral force at load increment j.
V_1	The total applied lateral force at the first increment of lateral load.
V_y	The effective yield strength determined from a bilinear curve fitted to the capacity curve according to Sec. A5.2.3.
W	See Sec. 1.1.5.
w_i	See Sec. 4.1.4.
Δ	The design story drift as determined in Sec. A5.2.6.
γ_i	The deformations for member i.
δ_j	The displacement of the control point at load increment j.
δ_T	The target displacement of the control point, determined according to Sec. A5.2.5.
δ_1	The displacement of the control point at the first increment of lateral load.
δ_y	The effective yield displacement of the control point determined from a bilinear curve fitted to the capacity curve according to Sec. A5.2.3.
ϕ_i	The amplitude of the shape vector at Level i, determined according to Sec. A5.2.4.
Ω_0	See Sec. 4.1.4.

A5.2 NONLINEAR STATIC PROCEDURE

Where the nonlinear static procedure is used to design structures, the requirements of this section shall apply.

A5.2.1 Modeling. A mathematical model of the structure shall be constructed to represent the spatial distribution of mass and stiffness of the structural system considering the effects of component nonlinearity for deformation levels that exceed the proportional limit. P-Delta effects shall be included in the analysis.

For regular structures with independent orthogonal seismic-force-resisting systems, independent two-dimensional models shall be permitted to be used to represent each system. For structures having plan irregularities Types 4 and 5 as defined in Table 4.3-2 or structures without independent orthogonal systems, a three-dimensional model incorporating a minimum of three degrees of freedom for each level of the structure, consisting of translation in two orthogonal plan directions and torsional rotation about the vertical axis, shall be used. Where the diaphragms are not rigid compared to the vertical elements of the seismic-force-resisting system, the model should include representation of the diaphragm flexibility.

Unless analysis indicates that a component remains elastic, a nonlinear force deformation model shall be used to represent the stiffness of the component before onset of yield, the yield strength, and the stiffness properties of the component after yield at various levels of deformation. The properties of nonlinear component models shall be consistent with principles of mechanics or laboratory data. Properties representing component behavior before yield shall be consistent with the provisions of Sec. 5.3.1. Strengths of elements shall not exceed expected values considering material overstrength and strain hardening. The properties of elements and components after yielding shall account for strength and stiffness degradation due to softening, buckling, or fracture as indicated by principles of mechanics or test data. The model for columns should reflect the influence of axial load where axial loads exceed 15 percent of the compression strength. The structure shall be assumed to have a fixed base or, alternatively, it shall be permitted to use realistic assumptions with regard to the stiffness and load-carrying characteristics of the foundations, consistent with site-specific soil data and rational principles of engineering mechanics.

A control point shall be selected for each model. For structures without penthouses, the control point shall be at the center of mass of the highest level of the structure. For structures with penthouses, the control point shall be at the center of mass of the level at the base of the penthouse.

A5.2.2 Analysis. The structure shall be analyzed for seismic actions occurring simultaneously with the effects of dead load in combination with not less than 25 percent of the required design live loads, reduced as permitted for the area of a single floor. The lateral forces shall be applied at the center of mass of each level and shall be proportional to the distribution obtained from a modal analysis for the fundamental mode of response in the direction under consideration. The lateral loads shall be increased incrementally in a monotonic manner.

At the j-th increment of lateral loading, the total lateral force applied to the model shall be characterized by the term V_j . The incremental increases in applied lateral force should be in steps that are sufficiently small to permit significant changes in individual component behavior (such as yielding, buckling or failure) to be detected. The first increment in lateral loading shall result in linear elastic behavior. At each analysis step, the total applied lateral force, V_j, the lateral displacement of the control point, δ_j, and the forces and deformations in each component shall be recorded. The analysis shall be continued until the displacement of the control point is at least 150 percent of the target displacement determined in accordance with Sec. A5.2.5. The structure shall be designed so that the total applied lateral force does not decrease in any analysis increment for control point displacements less than or equal to 125 percent of the target displacement.

A5.2.3 Effective yield strength and effective period. A bilinear curve shall be fitted to the capacity curve, such that the first segment of the bilinear curve coincides with the capacity curve at 60 percent of the effective yield strength, the second segment coincides with the capacity curve at the target displacement, and the area under the bilinear curve equals the area under the capacity curve, between the origin and the target displacement. The effective yield strength, V_y, corresponds to the total applied lateral force at the intersection of the two line segments. The effective yield displacement, δ_y, corresponds to the control point displacement at the intersection of the two line segments.

The effective fundamental period, T_e, shall be determined using Eq. A5.2-1 as follows:

$$T_e = T_1\sqrt{\frac{V_1/\delta_1}{V_y/\delta_y}} \tag{A5.2-1}$$

where V_1, δ_1, and T_1 are determined for the first increment of lateral load.

A5.2.4 Shape vector. The shape vector shall be equal to the first mode shape of the structure in the direction under consideration, determined by a modal analysis of the structure at the first increment of lateral load, and normalized to have unit amplitude at the level of the control point. It shall be permitted to substitute the deflected shape of the structure at the step at which the control point displacement is equal to the effective yield displacement in place of the mode shape, for determination of the shape vector.

A5.2.5 Target displacement. The target displacement of the control point, δ_T, shall be determined using Equation A5.2-2 as follows:

$$\delta_T = C_0 C_1 S_a \left(\frac{T_e}{2\pi}\right)^2 g \tag{A5.2-2}$$

where the spectral acceleration, S_a, is determined from either Sec. 3.3.4 or Sec. 3.4.4 at the effective fundamental period, T_e, g is the acceleration of gravity, and the coefficients C_0 and C_1 are determined as follows.

The coefficient C_0 shall be calculated using Equation A5.2-3 as:

$$C_0 = \frac{\sum_{i=1}^{n} w_i \phi_i}{\sum_{i=1}^{n} w_i \phi_i^2} \tag{A5.2-3}$$

where:

w_i = the portion of the seismic weight, *W*, at Level *i*, and

ϕ_i = the amplitude of the shape vector at Level *i*.

Where the effective fundamental period of the structure in the direction under consideration, T_e, is greater than T_s, as defined in Sec. 3.3.4 or Sec. 3.4.4, the coefficient C_1 shall be taken as 1.0. Otherwise, the value of the coefficient C_1 shall be calculated using Eq. A5.2-4 as follows:

$$C_1 = \frac{1}{R_d}\left(1 + \frac{(R_d - 1)T_s}{T_e}\right) \tag{A5.2-4}$$

where R_d is given by Eq. A5.2-5 as follows:

$$R_d = \frac{S_a}{V_y / W} \tag{A5.2-5}$$

and T_s and V_y are defined above, S_a is the design spectral acceleration at the effective fundamental period, T_e, and *W* is defined in Sec. 5.2.

A5.2.6 Story drift. The design story drift, Δ, taken as the value obtained for each story at the step at which the target displacement is reached shall not exceed the drift limit specified in Sec. 4.5.1 multiplied by $0.85R/C_d$.

A5.2.7 Member strength. In addition to satisfying the requirements of this Appendix, member strengths also shall satisfy the requirements of Sec. 4.2.2 using $E = 0$, except that Section 4.2.2.2 shall apply where these *Provisions* specifically require the consideration of structural overstrength on the design seismic force.

Where these *Provisions* require the consideration of structural overstrength according to Sec. 4.2.2.2, the value of the individual member forces, Q_{Ei} obtained from the analysis at the target displacement shall be taken in place of the quantity $\Omega_0 Q_E$.

A5.2.8 Distribution of design seismic forces. The lateral forces used for design of the members shall be applied at the center of mass of each level and shall be proportional to the distribution obtained from a modal analysis for the fundamental mode of response in the direction under consideration.

A5.2.9 Detailed evaluation. Sec. A5.2.9.1 and Sec. A5.2.9.2 need not be satisfied if the effective yield strength exceeds the product of the system overstrength factor as given in Table 4.3-1 and the seismic base shear determined in Sec. 5.2.1, modified to use the effective fundamental period T_e in place of *T* for the determination of C_s.

A5.2.9.1 Required member force and deformation. For each nonlinear static analysis the design response parameters, including the individual member forces, Q_{Ei}, and member deformations, γ_i, shall be taken as the values obtained from the analysis at the step at which the target displacement is reached.

A5.2.9.2 Member. The adequacy of individual members and their connections to withstand the member forces, Q_{Ei}, and member deformations, γ_i, shall be evaluated based on laboratory test data for similar components. The effects of gravity and other loads on member deformation capacity shall be considered in these evaluations. The deformation of a member supporting gravity loads shall not exceed

(i) two-thirds of the deformation that results in loss of ability to support gravity loads, and (ii) two-thirds of the deformation at which the member strength has deteriorated to less than the 70 percent of the peak strength of the component model. The deformation of a member not required for gravity load support shall not exceed two-thirds of the value at which member strength has deteriorated to less than 70% of the peak strength of the component model. Alternatively, it shall be permissible to deem member deformation to be acceptable if the deformation does not exceed the value determined using the acceptance criteria for nonlinear procedures given in the *Prestandard and Commentary for the Seismic Rehabilitation of Buildings* (FEMA 356) for the Life Safety performance level.

Member forces shall be deemed acceptable if not in excess of expected capacities.

A5.2.10 Design review. An independent team composed of at least two members, consisting of registered design professionals in the appropriate disciplines and others, with experience in seismic analysis methods and the theory and application of nonlinear seismic analysis and structural behavior under earthquake loading, shall perform a review of the design of the seismic force resisting system and the supporting structural analyses. The design review shall include (i) review of any site-specific seismic criteria employed in the analysis including the development of site-specific spectra, and (ii) review of the determination of the target displacement and effective yield strength of the structure.

For those structures with effective yield strength less than the product of the system overstrength factor as given in Table 4.3-1 and the seismic base shear determined in Sec. 5.2.1, modified to use the effective fundamental period T_e in place of T for the determination of C_s, the design review shall further include, but need not be limited to, the following:

1. Review of acceptance criteria used to demonstrate the adequacy of structural elements and systems to withstand the calculated force and deformation demands, together with that laboratory and other data used to substantiate such criteria. Review of the acceptance criteria for nonlinear procedures given in the *Prestandard and Commentary for the Seismic Rehabilitation of Buildings* (FEMA 356) shall be at the discretion of the design review team.
2. Review of the final design of the entire structural system and all supporting analyses.

The design review team shall issue a report that identifies, within the scope of the review, significant concerns and any departures from general conformance with the *Provisions.*

REFERENCES

ATC 40 (SSC, 1996) *Seismic Evaluation and Retrofit of Concrete Buildings,* SSC Report No. 96-01, Seismic Safety Commission, State of California, Sacramento, California. Developed by the Applied Technology Council, Redwood City, California.

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HAZUS (NIBS, 1999), *HAZUS99 Technical Manual,* National Institute of Building Science, Washington, D.C. Developed by the Federal Emergency Management Agency through agreements with the National Institute of Building Sciences.